**K21P 1072**

III Semester M.Sc. Degree (CBSS – Reg./Suppl./Imp.)
Examination, October 2021
(2018 Admission Onwards)
MATHEMATICS
MAT 3E01 : Graph Theory

Time : 3 Hours                                                                 Max. Marks : 80

*Instructions* : 1) *Answer* **any 4** *questions from Part –* **A. Each** *question carries* **four** *marks.*

2) *Answer* **any 4** *questions from Part –* **B** *without omitting* **any** *Unit.* **Each** *question carries* **16** *marks.*

PART – A

I. Answer **any 4** questions. **Each** question carries **4** marks.

1) Let $(S_1, S_2, .... S_n)$ be any partition of the set of integers $\{1, 2, ... r_n\}$, then prove that for some i, Si contains three integers x, y and z satisfying the equation $x + y = z$.

2) Let G be a k-critical graph with a 2-vertex cut {u, v}. Then prove that $d(u) + d(v) \geq 3k - 5$.

3) When do you say that a graph G is embeddable on a surface S ? Further prove that a graph G is embeddable in the plane if and only if it is embeddable on the sphere.

4) If two bridges overlap, then prove that either they are skew or else they are equivalent 3-bridges.

5) Prove that a tree has atmost one perfect matching.

6) Prove that a simple graph G is n-connected if and only if given any pair of distinct vertices u and v of G, there are at least n-internally disjoint paths from u to v.

P.T.O.

## PART – B

### Unit – I

II. a) If G is simple and contains no $k_{m+1}$, then prove that $\sum(G) \leq \sum(T_{m,v})$, $T_{m,v}$ denote the complete m partite graph on v vertices in which all parts are as equal in size as possible. Also prove that $\sum(G) = \sum(T_{m,v})$ only if $G \cong T_{m,v}$.

b) Define a k-critical graph and if G is a k-critical graph, then show that $\delta \geq k-1$.

III. Define the Ramsey number r(k, l) and find an upper bound and lower bound for the Ramsey number r(k, k).

IV. a) If G is simple, then prove that $\pi_k(G) = \pi_k(G-e) - \pi_k(G \cdot e)$ for any edge e of G.

b) For any graph G, prove that $\pi_k(G)$ is a polynomial in k of degree v with integer coefficients, leading term $k^v$ and constant term zero. Further prove that the coefficients of $\pi_k(G)$ alternate in sign.

### Unit – II

V. a) Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

b) If G is bipartite and if $p \geq \Delta$, then prove that there exist p-disjoint matchings $M_1, M_2, \ldots M_p$ of G such that $E = M_1 \cup M_2 \cup \ldots \cup M_p$. Also show that any two matchings $M_i$ and $M_j$ differ in size by at most one.

VI. a) Describe a good algorithm for finding a proper $\Delta$-edge colouring of a bipartite graph G.

b) If G is simple, then prove that either $X'(G) = \Delta$ or $X'(G) = \Delta + 1$.

VII. a) Show that $K_5$ can be embedded on the torus and $K_{33}$ on the Mobius band.

b) State and prove Euler's formula for planar graphs and show that $K_{3,3}$ is not planar. Also check the planarity of $K_{33} - e$.

## Unit – III

VIII. a) Let G be a bipartite graph with bipartition (X, Y). Then prove that G contain a matching that saturates every vertex in X if and only if $|N(S)| \geq |S|$ for all $S \leq X$. 10

b) State and prove the marriage theorem. 6

IX. a) Give the Kuhn-Munkres algorithm to find an optimal matching in a weighted complete bipartite graph. Also draw its flow chart. 10

b) Let $l$ be a feasible vertex labelling of G. If $G_l$ contains a perfect matching $M^*$, then $M^*$ is an optimal matching of G. 6

X. a) Let u and v be two non-adjacent vertices of a graph G. Then prove that the maximum number of internally disjoint u-v paths in G equals the minimum number of vertices in a u-v separating set. 8

b) Let G be a simple graph, then prove that $K(G) \leq K_e(G) \leq \delta(G)$. 8

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