



K21P 1072

III Semester M.Sc. Degree (CBSS – Reg./Suppl./Imp.)

Examination, October 2021

(2018 Admission Onwards)

MATHEMATICS

MAT 3E01 : Graph Theory

Time : 3 Hours

Max. Marks : 80

Instructions : 1) Answer **any 4** questions from Part – A. **Each** question carries **four** marks.

2) Answer **any 4** questions from Part – B without omitting **any** Unit. **Each** question carries **16** marks.

PART – A

I. Answer **any 4** questions. **Each** question carries 4 marks.

- 1) Let (S_1, S_2, \dots, S_n) be any partition of the set of integers $\{1, 2, \dots, r_n\}$, then prove that for some i , S_i contains three integers x, y and z satisfying the equation $x + y = z$.
- 2) Let G be a k -critical graph with a 2-vertex cut $\{u, v\}$. Then prove that $d(u) + d(v) \geq 3k - 5$.
- 3) When do you say that a graph G is embeddable on a surface S ? Further prove that a graph G is embeddable in the plane if and only if it is embeddable on the sphere.
- 4) If two bridges overlap, then prove that either they are skew or else they are equivalent 3-bridges.
- 5) Prove that a tree has atmost one perfect matching.
- 6) Prove that a simple graph G is n -connected if and only if given any pair of distinct vertices u and v of G , there are at least n -internally disjoint paths from u to v .

P.T.O.

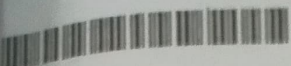
PART - B

Unit - I

- II. a) If G is simple and contains no K_{m+1} , then prove that $\sum(G) \leq \sum(T_{m,v})$,
 $T_{m,v}$ denote the complete m partite graph on v vertices in which all parts are as equal in size as possible. Also prove that $\sum(G) = \sum(T_{m,v})$ only if $G \cong T_{m,v}$.
- b) Define a k -critical graph and if G is a k -critical graph, then show that $\delta \geq k-1$.
- III. Define the Ramsey number $r(k, l)$ and find an upper bound and lower bound for the Ramsey number $r(k, k)$.
- IV. a) If G is simple, then prove that $\pi_k(G) = \pi_k(G - e) - \pi_k(G \cdot e)$ for any edge e of G .
- b) For any graph G , prove that $\pi_k(G)$ is a polynomial in k of degree v with integer coefficients, leading term k^v and constant term zero. Further prove that the coefficients of $\pi_k(G)$ alternate in sign.

Unit - II

- V. a) Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.
- b) If G is bipartite and if $p \geq \Delta$, then prove that there exist p -disjoint matchings M_1, M_2, \dots, M_p of G such that $E = M_1 \cup M_2 \cup \dots \cup M_p$. Also show that any two matchings M_i and M_j differ in size by at most one.
- VI. a) Describe a good algorithm for finding a proper Δ -edge colouring of a bipartite graph G .
- b) If G is simple, then prove that either $X'(G) = \Delta$ or $X'(G) = \Delta + 1$.
- VII. a) Show that K_5 can be embedded on the torus and K_{33} on the Mobius band.
- b) State and prove Euler's formula for planar graphs and show that $K_{3,3}$ is not planar. Also check the planarity of $K_{33} - e$.



Unit – III

- VIII. a) Let G be a bipartite graph with bipartition (X, Y) . Then prove that G contain a matching that saturates every vertex in X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$. 10
- b) State and prove the marriage theorem. 6
- IX. a) Give the Kuhn-Munkres algorithm to find an optimal matching in a weighted complete bipartite graph. Also draw its flow chart. 10
- b) Let l be a feasible vertex labelling of G . If G contains a perfect matching M^* , then M^* is an optimal matching of G . 6
- X. a) Let u and v be two non-adjacent vertices of a graph G . Then prove that the maximum number of internally disjoint u - v paths in G equals the minimum number of vertices in a u - v separating set. 8
- b) Let G be a simple graph, then prove that $K(G) \leq K_e(G) \leq \delta(G)$. 8
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